Project Report: 2018

The Role of Reasoning-and-Proving in High Schools: Examining Curriculum Materials and Teacher Perceptions

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ABSTRACT

The proposed study is a qualitative analysis of the reasoning-and-proving opportunities contained in South African curriculum materials for high school geometry and exploratory interviews with geometry teachers about their perceptions of the role of reasoning-and-proving in the high school curriculum. The South African context provides a rich opportunity to study these topics because, in recent years, they removed geometry from the official curriculum but have now reinstated it. For this reason, some mathematics teachers have experience teaching reasoning-and-proving outside of geometry (a rarity in the United States) and are also likely to have formed strong ideas about the desired role of reasoning-and-proving in the mathematics curriculum. Dr. Otten has published multiple studies that established frameworks for this analysis and Dr. Govender provides mathematical knowledge and awareness of the unique aspects of the South African context of this study.

The following research questions are guiding the study:

1. What is the nature and extent of reasoning-and-proving opportunities contained in secondary-level geometry textbooks?
2. What types of reasoning-and-proving opportunities do secondary-level mathematics teachers value for the students and how do they describe their desired role for reasoning-and-proving in their mathematics classes?

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To support Dr. Otten's travel to the University of the Western Cape to collaborate with Dr. Govender, we requested $4,415 from the UMSAE UM/UWC Linkage Program, with $1,080 matched by Dr. Otten's endowed research fund. This report will detail the activities that have occurred thus far due to that request being granted. The report is structured in three sections: Preparatory activities, activities during Dr. Otten's travel to the Western Cape, and continuing activities.

Preparatory Activities

The study of reasoning-and-proving opportunities in South African schools, particularly in relation to geometry topics, had two major prongs. The first was a textbook analysis of the types of reasoning-and-proving opportunities that are afforded to students. The second was interviews with mathematics teachers from a variety of school contexts in the Cape Town area. In advance of Dr. Otten's visit to the Western Cape, preparations were made by both Dr. Otten and Dr. Govender in relation to both prongs of the study.

For the textbook analysis, Dr. Otten gathered together his previously-developed reasoning-and-proving analytic framework (Otten, Gilbertson, Males, & Clark, 2014; Otten, Males, & Gilbertson, 2014) and updated the coding spreadsheets for said framework. He also reviewed more recent articles that had been published relating to reasoning-and-proving in textbooks (e.g., Hunte, 2018; Komatsu et al., 2017). Dr. Govender secured physical copies of four major geometry textbooks in use in Western Cape province schools. He also made scanned copies of the textbook sections relevant for reasoning-and-proving so that analytic mark-ups would be possible.

For the teacher interviews, Dr. Otten prepared semi-structured interview protocols for individual teacher interviews and for focus group interviews (see appendix). Dr. Govender reached out to schools in various quintiles and arranged for meeting times during Dr. Otten's visit so that the interviews could be conducted efficiently and in a short timeframe. Both Dr. Otten and Dr. Govender also secured IRB (#2011991) and ethical (HS18/3/22) clearance at their respective institutions for the interview study with human subjects.

Dr. Otten also worked with Dr. Uphoff and Ashley Rhode to make all the necessary arrangements for international travel. He also read several articles and book chapters on the history and current state of mathematics education in South Africa (e.g., Webb & Roberts, 2017).

Activities During Dr. Otten's Travel to the Western Cape

Travel dates: *July 23, 2018 – August 10, 2018*

Here is a photograph from Dr. Otten's arrival, with Dr. Govender, the Dean of the Faculty of Education (Dr. Nomlomo), the department chair (Dr. Khuzwayo), and Dr. Otten.

Research activities. Dr. Otten and Dr. Govender reviewed the structure and layout of the selected geometry textbooks, which included *Classroom Mathematics* (Grades 10, 12), *Via Afrika Mathematics* (Grade 11), *CAPS* (Grades 10, 12), *CAPS Platinum* (Grades 10, 11, 12), and *Mind Action Series Mathematics* (Grades 10, 11, 12). These textbooks were selected because they are commonly used in the province and the grade levels were selected based on the placement of geometry topics relevant to reasoning-and-proving opportunities. Dr. Govender also acclimated Dr. Otten to the official standards and expectations for secondary mathematics in the province. While on the campus of UWC, Dr. Otten began the textbook analysis to verify that the coding schemes, developed for use with U.S. geometry textbooks, were appropriate for use in the South African context. The types of reasoning-and-proving opportunities in the South African textbooks were similar to those found in other countries and the coding schemes were found to be appropriate.

The primary research activity during the visit was interview data collection. Dr. Otten and Dr. Govender conducted 10 individual interviews with mathematics teachers and 5 focus group interviews with a total of 22 mathematics teachers (some of whom also participated in individual interviews). Participating teachers came from the following schools, which represent a diverse cross-section of schools in the Cape Town area:

- Bellville South
- COSAT (Centre of Science and Technology)
- Excelsior
- Rustenburg Girls
- South Peninsula

Following interviews, Dr. Otten and Dr. Govender would meet to discuss emergent themes and write analytic memos based on initial impressions. Dr. Otten also used his endowed research

fund to pay for interview transcript, completed within 3-4 days of audio file submission (using Datagain Services). In this way, Dr. Otten and Dr. Govender could review some of the transcripts while Dr. Otten was still in the Western Cape, though detailed analysis would not occur until later.

Dr. Otten was also able to informally observe several of the teachers' instruction, but this was for his own background information and is not officially part of the analysis. Here are some photographs from school visits (permission granted by those depicted).

Dissemination and network-building activities. Dr. Otten was invited to give two presentations to faculty, students, and administrators at UWC. The first presentation was focused on his research related to reasoning-and-proving and was tailored for mathematics education faculty and mathematics faculty, who were invited over from the Faculty of Natural Science. The title of the first presentation was "Reasoning-and-Proving Opportunities in Secondary Textbooks: General and Particular Divisions." Dr. Govender was especially pleased that several faculty from mathematics attended this presentation and actively engaged with the topic and with the mathematics education faculty.

The second presentation was broadly oriented toward the full Faculty of Education. It focused on flipped instruction and the NSF-funded study by Dr. Otten and Dr. de Araujo from Mizzou focused on secondary algebra. The attendance at this seminar was above average and Dr. Otten received a substantial amount of positive feedback, including some contacts with scholars from UWC who are interested in learning more about the effectiveness of flipped instruction or who have taken some of the insights shared and are using them to refine their own use of instructional videos.

These presentations were a beneficial way to share with the UWC community some of the mathematics-education scholarship that is taking place at Mizzou, and so it is helping to maintain connections between the two scholarly communities. Below are pictures from the second seminar, which was entitled "Examining the Use of Instructional Videos as a Factor in Secondary Maths Instruction."

Dr. Otten was also able to use his visit to help disseminate scholarship from UWC to the U.S. and other parts of the world. In particular, he recorded interviews with two UWC faculty members—Bheki "Herbert" Khuzwayo and Bruce May—and released them as episodes of the Math Ed Podcast (www.mathedpodcast.com). This podcast is freely accessible and is listened to by mathematics scholars and mathematics teachers around the world. Individual episodes are, on average, downloaded or streamed by 1,300 people and so this brings new levels of exposure to the work of Dr. Khuzwayo and Dr. May.

Dr. Otten was also pleased to make connections with practitioners, not only the teachers in the study but also an elementary school where he received a full tour. And Dr. Otten also reconvened with Dr. Corvell Cranfield, a former colleague from graduate school at Michigan State University, who now works at the National Education Collaboration Trust in South Africa. He is part of a team that works to revitalize mathematics instruction in the bottom-quintile schools in the townships around Cape Town. The visit also helped facilitate connections among others, aside from Dr. Otten. For example, Dr. Otten was able to introduce (via email) a literacy education scholar from UWC and Drs. Kuby and Zapata from Mizzou. He was also able to put Dr. Khuzwayo from UWC in touch with Dr. Munter from Mizzou because both scholars are interested in the presence of racism in mathematics education. In both of these instances of new connections, there is interest in possibly pursuing a future UMSAE faculty travel exchange.

Finally, another network-building activity was Dr. Otten joining Dr. Govender's weekend summit at Gordon's Bay of mathematics teacher educators from across South Africa. This two-day summit involved discussions of the curriculum for teacher preparation related to geometry. Dr. Otten was an active participant in the discussions and was able to share some of the curricular features of teacher education in the U.S. and also learned a great deal about teacher education in South Africa. Below is a photograph of the teacher educators in attendance.

Continuing Activities

The primary activity since the trip has been data analysis and preliminary writing. Dr. Otten has continued the textbook analysis and is on pace to complete it within the month of January 2019, at which point they will begin preparation of a manuscript on the nature of reasoning-and-proving opportunities in those textbooks, which can be compared with similar analyses in other countries. Dr. Otten and Dr. Govender have also conducted qualitative thematic analysis of the interview data and are preparing a research manuscript about the teachers' perspectives on reasoning-and-proving. Particular themes that have emerged are that the teachers hold differing views on the scope and purpose of reasoning-and-proving in secondary schools. Some teachers feel that reasoning-and-proving should be solely tied to geometry topics, whereas others view reasoning-and-proving as a mode of thinking that can apply to all topics in mathematics. With regard to students, some teachers expressed that reasoning-and-proving is a challenging practice that is best taught to only the high-performing students, whereas other teachers expressed that reasoning-and-proving is an aspect of critical thinking that should be made accessible to all students. Dr. Otten shared preliminary findings from the interview data at a weekly seminar among the mathematics education group at Mizzou, and he solicited feedback about the analysis from his colleagues. Dr. Otten and Dr. Govender hope to finalize their manuscript and submit it for publication within the month of February 2019. Dr. Otten is also going to offer to hold a session at the Mizzou Ed Bridge (a common space in the College of Education for sharing work that relates to diversity and equity) in Spring 2019 to describe his experiences in the Western Province and raise some issues related to inequities in mathematics education that are prevalent in both South Africa and the United States.

As Dr. Otten and Dr. Govender complete the two manuscripts related to this research study, they also fully intend to identify potential funding sources to continue working together. In particular, they may pursue research related to the teaching and learning of more specific geometry topics and geometry is a troublesome area with regard to student achievement in both South Africa and the United States.

Acknowledgments

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APPENDIX: Interview Protocol

Focus Group

Getting Started

- Thank you. We are trying to learn about your experiences with teaching mathematics and your opinions on some issues in mathematics education. You are the experts on your particular students and on what it's like in your classrooms.
- Your identities will be protected. But to get started, **please share the following:**
    - **Name, Number of Years Teaching, Courses You've Taught Recently**.
- This is meant to be a conversation, so speak freely. And please respond, add on, or clarify at any time.
- **Is it important for Grade 10, 11, and 12 students to learn mathematics? Why or why not?**
    - Can you think of any particular students where you felt it was especially beneficial to them that they learned mathematics?
    - Are there any students where you felt it was not especially important or not appropriate for them to learn mathematics? In other words, does *every* student need to learn Grade 10–12 maths?
- We are curious about your perspectives on reasoning-and-proving, and by reasoning-and-proving we mean the broad process of determining the truth of mathematical results. So reasoning-and-proving includes conjecturing, attempting to justify claims, using logic, critiquing and refining arguments, and ultimately it includes formal proof.
- **Is it important for Grade 10, 11, and 12 students to learn reasoning-and-proving? Why or why not?**
    - Does *every* student need to learn reasoning-and-proving?

Approach to Reasoning-and-Proving

- Starting with those who teach the earliest grade and then moving forward…
    - When the students come to you, what experiences have they already had with reasoning-and-proving?
    - What do you try to emphasize with regard to reasoning-and-proving? (Conjecturing, justifying, logic, critiquing and revising, formal proof) And it is fine to say that you don't emphasize it very much, because perhaps it is not in every grade.
- What factors influence the extent to which or the way that you teach reasoning-and-proving: standards, textbooks, past precedent, colleagues, personal preference, students' needs, or something else?
- What are some of the most challenging aspects of teaching proof?
- Should students conjecture the mathematical claims that they will prove or should the conjectures be provided to them (e.g., from the textbook or teacher)?

- Should students learn a particular format for writing proofs or should there be some flexibility in proof format?
- Should students be held to mathematically rigorous standards of formal proof or should there be some leeway in terms of justifications and explanations that make sense to the students?

Proof in Algebra and Geometry

- What do you remember about the changes in maths standards from 2006 to 2012?
    - How many of you were teaching maths for at least a portion of that time between 2006 and 2012? (Record number.)
    - For those of you who were teaching at that time, how did you teach reasoning-and-proving then? How does it compare or contrast with how you teach it now?
- Should reasoning-and-proving be taught primarily in conjunction with geometry topics, primarily in conjunction with other topics (like algebra or trigonometry), or should reasoning-and-proving be taught consistently across all topics?

Individual

Background Questions

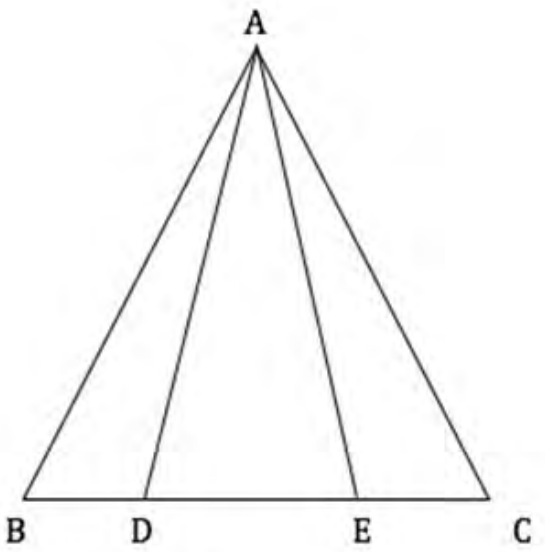
- Name.
- Tell me about your teacher preparation program. Your mathematical background.
- How many years have you been teaching? In what contexts?
- For how long have you taught proof in mathematics?
- With regard to proof, what were your experiences like as a *learner*? In high school? In college?

Approach to Reasoning-and-Proving

- Do you think all students should learn proof? Why or why not?
- How do you usually introduce students to proof?
    - Do you explicitly address rules of logic? Do you promote a particular form of proof writing (e.g., two-columns, flow charts)?
- Describe one of your favorite or most memorable lessons about proof.
- What are some of the most challenging aspects of teaching proof?
- What textbook or curriculum $_{\text{resources}}$ do you use to teach proof? Can you show me or talk through how you use the textbook during your planning? During your lesson enactment?

Proof in Algebra and Geometry

- (Share sample proof problems from geometry) Would you typically expect your students to solve problems like this?
    - If no, why not? If yes, what sorts of responses would you expect?
- (Share sample proof problems from algebra) Would you typically expect your students to solve problems like this?
    - If no, why not? If yes, what sorts of responses would you expect?
- What do you see as the advantages and disadvantages of teaching proof along with geometry topics? along with algebra topics?



Given: $\overline{AB} = \overline{AC}$ $< BAD \cong < CAE$

Prove: Triangle ADE is isosceles.

CLAIM:

If two angles of a triangle are congruent, then the triangle is isosceles.

This claim is (circle one)

ALWAYS TRUE

SOMETIMES TRUE

NEVER TRUE

Prove that your answer is correct.

CLAIM:

If a polynomial has $(x - k)$ *as a factor, then k is a root of the polynomial.*

This claim is (circle one)

ALWAYS TRUE

SOMETIMES TRUE

NEVER TRUE

Prove that your answer is correct.

Given: $3x^2 - 24x + 48 = 0$

Prove: $x = 4$